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Non-Newtonian Viscosity Modeling of Crude Oils—Comparison Among Models

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Abstract The presence of precipitated wax or even just low temperatures may induce non-Newtonian rheological behavior in crude oils. Such behavior can be found at operating conditions, for instance, in reservoirs at deep-water conditions. Therefore, reliable rheological models for crude oils applicable over the wide range of conditions the fluid may encounter are essential for a large number of oil technology applications. Such models must also be composition dependent, as many applications require predicting the rheological behavior of the fluid under strong compositional changes, e.g., recovery applications such as vapor extraction (VAPEX) processes or blending of fluids for improved rheological characteristics for piping, among many other applications. In this study, a comparative analysis between some published models applicable to the description of the non-Newtonian behavior of crude oils is carried out. Emphasis is placed on the stability of the model predictions within the wide range of conditions that may be encountered.

Keywords Friction theory · Non-Newtonian · Rheology · Viscosity · Waxy oils

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1 Introduction

A large number of oil technology applications need reliable rheological models over the wide range of conditions a reservoir fluid may encounter in order to predict, design, or simulate flow behavior with good accuracy. For example, strong variations in temperature may induce non-Newtonian rheological responses for waxy oils if temperature decreases below the "Wax Appearance Temperature" (WAT) or for heavy oils at ambient or deep-water conditions. Changes in composition may also affect the capability of viscosity predictions of some models in the Newtonian or non-Newtonian response. Vapor extraction (VAPEX) processes are an example where heavy oils may undergo important variations in temperature and composition and therefore in their viscosity.

Based on previous applications of the friction theory (FT) to reservoir fluids [1], a FT extended model capable of accurately describing the non-Newtonian rheological behavior of waxy oil mixtures has recently been developed [2]. This is a three-parameter non-Newtonian rheological model, which contains one viscosity scaling parameter (K_0), one low-temperature enhancement parameter (T_s), and one rate of shearing dissipation parameter (γ_0). In this study, a comparative analysis is carried out between the FT model and three other types of models also applicable to the description of non-Newtonian behavior: the model by Pedersen and Rønningsen [3], the Al-Zahrani model [4] used by Dante et al. [5], and a simple power-law fit with temperature-dependent parameters.

2 Non-Newtonian Friction Theory Modeling for Crude Oils

For oil applications the one-parameter FT general model [6] has been successfully applied to cover, practically the full range of normally encountered reservoir applications [1]. In the case of the Newtonian FT model, the total viscosity η is separated into a dilute gas viscosity term, η_0 and a residual friction term, η_f:

$$\eta = \eta_0 + \eta_f \tag{1}$$

For many cases, particularly when dealing with reservoir fluids,η_0 can be reasonably well described with predictive empirical models such as the one proposed by Chung et al. [7]. The residual friction term η_f is given by

$$\eta_f = \eta_c \left(\hat{\kappa}_r \frac{p_r}{p_c} + \hat{\kappa}_a \frac{p_a}{p_c} + \hat{\kappa}_{rr} \frac{p_r^2}{p_c^2} \right) \tag{2}$$

where η_c is a characteristic fluid viscosity scaling parameter, p_a and p_r are the attractive and repulsive pressure contributions, and p_c is the critical pressure. The friction coefficients ($\hat{\kappa}_r$, $\hat{\kappa}_a$, and $\hat{\kappa}_{rr}$) depend on the reduced temperature only and can be represented by analytical functions whose parameters are, for a given EoS, universal constants. For oil applications, there must clearly be a related characterization procedure as well as applicable mixing rules, all of which are described elsewhere [1].

In the case of reservoir fluids, once the fluid has been properly characterized, tabulated values [6] for η_c are used for the well-defined light compounds (i.e., methane, ethane, etc.). However, the heavy fraction (C_7+) is separated into pseudocomponents, and the viscosity characterization parameters are obtained from the modified Uyehara–Watson [8] equation:

$$\eta_{\mathrm{c,i}} = K_\mathrm{c} \frac{\sqrt{M_\mathrm{i}}\, p_{\mathrm{c,i}}^{2/3}}{T_{\mathrm{c,i}}^{1/6}} \tag{3}$$

where M, p_c, and T_c are the molar mass, critical pressure, and critical temperature, respectively, for the pseudocomponent "i", while K_c is suggested as an adjustable global parameter characteristic of the particular fluid being analyzed. This parameter is tuned to the available experimental viscosity measurements (at least one measurement is required) or it can be predicted using related correlations [9].

In a previous article [2], the FT model was modified to account for non-Newtonian behavior, particularly in waxy oils. The FT model used in this study is essentially the same as in the previous study, with the exception of a small algebraic modification in order to make nearly all the parameters dimensionless. In the FT model, it is assumed that all of the fluid's light fraction remains Newtonian, while the non-Newtonian behavior is described as a departure from the Newtonian characterization and modeling recommended for the heavy fraction. The following model describes this departure in terms of the K_c constant in Eq. 3:

$$\begin{aligned} K_\mathrm{c} = \; & K_0\left(1 + \frac{1}{\left(1+\gamma_0\dot{\gamma}^{0.7}\right)} \exp\left[\left(1 + \left(s_0\left(T_\mathrm{r}-T_\mathrm{s}\right)\right)^6\right)\left(s_1\left(T_\mathrm{r}-T_\mathrm{s}\right)\right)^3\right.\right. \\ & \left.\left. + \frac{s_2}{\left(T_\mathrm{r}-T_\mathrm{s}\right)^{0.5}}\right]\right) \end{aligned} \tag{4}$$

where $\dot{\gamma}$ is the shear rate given in s^{-1}, γ_0 is a shear rate parameter with units of $\mathrm{s}^{0.7}$, T_s is an adjustable reduced temperature related to a reduced WAT, and T_r is the fluid reduced temperature,

$$T_\mathrm{r} = \frac{T}{T_{\mathrm{c}+}} \tag{5}$$

Here, the reducing temperature $T_{\mathrm{c}+}$ is calculated with the same empirical equation used in the characterization procedure of the heavy fraction [1]:

$$T_{\mathrm{c}+} = -423.587 + 210.152 \ln\left[\frac{M^+}{M_\mathrm{ref}}\right] \tag{6}$$

where $T_{\mathrm{c}+}$ is given in K and M^+ is the total molar mass of the heavy fraction (C_{7+}) in $\mathrm{g}\cdot\mathrm{mol}^{-1}$; the reference value of M_ref is set to $1\ \mathrm{g}\cdot\mathrm{mol}^{-1}$.

In spite of its simplicity, Eq. 4 introduces several important features that are typical for a wide range of non-Newtonian fluids.

2.1 Newtonian Plateaus

It should be observed that as $\dot{\gamma} \to 0$, K_c approaches a temperature-dependent parameter, where as $\dot{\gamma} \to \infty$, it approaches the constant in the original Newtonian model, Eq. 3. This will generate two Newtonian plateaus: one at a low rate of shearing, while the other will be found in the region of high shear rates.

2.2 Shear-Thinning Region

A shear-thinning region is found between the two Newtonian plateaus. This region is governed by the 0.7 exponent in the $\left(1 + \gamma_0 \dot{\gamma}^{0.7}\right)$ term of the denominator of Eq. 4. The exponent 0.7 is consistent with the physics observed in other relevant rheological systems, such as polymer blends [10], although it can also be considered an additional adjustable parameter.

2.3 Low-Temperature Enhancement

In general, as the temperature of a fluid approaches that of solid formation, such as the WAT or the triple point in the case of pure substances, the fluid will show a significant viscosity enhancement. The exponential term is an empirical relation that is able to reasonably well represent the behavior of the studied waxy oils at low temperature. The temperature correction term depends on three dimensionless parameters (s_0, s_1, s_2) which were correlated against published data on waxy oils [3]. The suggested correlations for the low-temperature model parameters are of the form:

$$s_0 = -26.219 \left(\frac{K_0}{K_{\text{ref}}}\right) + 46.718 \tag{7}$$

$$s_1 = -1.1684 \left(\frac{K_0}{K_{\text{ref}}}\right) + 0.28159 \tag{8}$$

$$s_2 = 4.03395 \left(\frac{\gamma_0}{\gamma_{\text{ref}}}\right) + 1.29209 \tag{9}$$

The K_{ref} is the Uyehara–Watson FT estimation for n-alkanes [11] corresponding to

$$K_{\text{ref}} = 0.79483 \left[\frac{\text{K}^{1/6}\text{mol}^{1/2}}{\text{g}^{1/2}\text{bar}^{2/3}} \mu\text{Pa} \cdot \text{s}\right] \tag{10}$$

and γ_{ref} is set to 1 $\text{s}^{0.7}$. The final results for the modeling of the eight selected wax oils are reported in Tables 1 and 2. Table 1 shows the characterization of the eight oils and all the values are the same as in the related previous study [2] while Table 2 shows the new regression values modified for the non-Newtonian model used in this work,

Table 1 Compositional characterization and characterization parameters for the heavy fractions (F1, F2, F3, F4) of selected waxy oils [3]

	Oil 1 CS(4.8) M(C7+): 229.1 g/mol				Oil 2 CS(3.5) M(C7+): 243.3 g/mol				Oil 3 CS(6.1) M(C7+): 272.5 g/mol				Oil 4 CS(5.9) M(C7+): 239.7 g/mol			
	Mole fraction (x)				Mole fraction (x)				Mole fraction (x)				Mole fraction (x)			
C1	0.0000				0.0000				0.0000				0.00281			
C2	0.0000				0.00111				0.00336				0.00448			
C3	0.0001				0.01219				0.01319				0.01171			
iC4	0.0004				0.00645				0.00566				0.00503			
nC4	0.00331				0.0282				0.01741				0.01198			
iC5	0.00601				0.01954				0.01086				0.00809			
nC5	0.01212				0.03324				0.01471				0.00933			
C6	0.03466				0.05529				0.0268				0.01989			
F1	0.403784				0.376825				0.392753				0.394100			
F2	0.257101				0.230969				0.242059				0.249493			
F3	0.177178				0.151912				0.169072				0.175174			
F4	0.105337				0.084274				0.104126				0.107913			
	T_c (K)	p_c (bar)	ω	M (g/mol)	T_c (K)	p_c (bar)	ω	M (g/mol)	T_c (K)	p_c (bar)	ω	M (g/mol)	T_c (K)	p_c (bar)	ω	M (g/mol)
F1	605.4	27.71	0.4539	133.83	609.2	27.39	0.4615	136.25	639.7	24.86	0.5275	157.52	616.2	26.79	0.4760	140.88
F2	700.3	20.42	0.6841	210.18	712.1	19.64	0.7189	222.29	741.4	17.81	0.8126	255.59	712.3	19.63	0.7196	222.53
F3	778.5	15.71	0.9465	304.10	800.1	14.58	1.0330	337.97	816.8	13.76	1.1047	365.92	786.6	15.28	0.9781	316.94
F4	887.8	10.71	1.4605	513.01	923.9	9.39	1.6774	609.22	918.7	9.57	1.6442	594.16	888.4	10.68	1.4639	514.48

Table 1 continued

	Oil 5 CS(4.3) M(C7+): 280.9 g/mol				Oil 6 CS(5.1) M(C7+): 310.7 g/mol				Oil 7 CS(6.4) M(C7+): 237.1 g/mol				Oil 8 CS(4.6) M(C7+): 234.4 g/mol			
	Mole fraction (x)				Mole fraction (x)				Mole fraction (x)				Mole fraction (x)			
C1	0.0000				0.0000				0.0000				0.0000			
C2	0.00085				0.00338				0.00075				0.00071			
C3	0.00924				0.01391				0.00462				0.01115			
iC4	0.00394				0.00502				0.00273				0.00589			
nC4	0.02191				0.01457				0.00895				0.02355			
iC5	0.01446				0.00849				0.00721				0.01512			
nC5	0.02574				0.01133				0.0116				0.0246			
C6	0.04299				0.01969				0.03009				0.04454			
F1	0.400011				0.420423				0.390375				0.377406			
F2	0.235559				0.243343				0.251987				0.238114			
F3	0.156046				0.163544				0.179339				0.162927			
F4	0.089254				0.096300				0.112349				0.095993			
	T_c (K)	p_c (bar)	ω	M (g/mol)	T_c (K)	p_c (bar)	ω	M (g/mol)	T_c (K)	p_c (bar)	ω	M (g/mol)	T_c (K)	p_c (bar)	ω	M (g/mol)
F1	635.8	25.17	0.5186	154.63	656.5	23.55	0.5674	170.66	617.6	26.67	0.4790	141.83	608.5	27.45	0.4601	135.79
F2	747.1	17.48	0.8319	262.58	771.4	16.09	0.9195	294.84	709.6	19.807	0.7115	219.72	705.3	20.09	0.6987	215.23
F3	833.6	12.98	1.1811	396.38	854.9	12.04	1.2848	438.70	781.1	15.57	0.9564	308.72	785.0	15.36	0.9718	314.55
F4	951.0	8.50	1.8581	693.01	966.2	8.03	1.9668	745.04	879.4	11.04	1.4135	492.81	896.286	10.39	1.5085	533.88

Table 2 Rheological FT model results

	Oil 1	Oil 2	Oil 3	Oil 4	Oil 5	Oil 6	Oil 7	Oil 8
K_0/K_{ref}	2.16514	0.94618	0.94346	1.05614	1.55913	1.21630	1.04989	1.22004
T_s	0.232934	0.251356	0.244583	0.250187	0.089124	0.206613	0.247086	0.226368
$\gamma_0(s^{0.7})$	0.309688	0.070528	0.222278	0.075298	0.143549	0.011334	0.006225	0.049270
AAD	19.15 %	13.58 %	18.62 %	12.98 %	9.86 %	6.20 %	15.18 %	12.95 %

i.e., K_0/K_{ref}, γ_0, and T_s, along with the resulting absolute average deviation (*AAD*),

$$\text{AAD} = \frac{1}{n}\sum_{i=1}^{n}\left|\frac{\eta_{\text{cal},i} - \eta_{\exp,i}}{\eta_{\exp,i}}\right| \tag{11}$$

We point out that the proposed FT non-Newtonian model reproduced the viscosities of all the studied waxy oils with an overall AAD better than 15 %. Another point to be underlined is the fact that a value of K_0/K_{ref} close to one implies a viscosity behavior similar to that of *n*-alkanes, i.e., a highly waxy oil.

3 The Pedersen and Rønningsen Model

The Pedersen and Rønningsen model [3] takes into account the amount of precipitated wax (solid phase) in the crude oil, which is assumed to be suspended in the fluid phase, producing non-Newtonian effects. In order to develop a model capable of reproducing the non-Newtonian viscosity behavior, the authors coupled Richardson's model [12] for the viscosity of oil–water emulsions and Casson's constitutive equation [13] for non-Newtonian viscosity showing yield stress phenomena. According to Pedersen and Rønningsen, waxy crude oils commonly exhibit a shear-rate dependent viscosity and yield stress. The resulting model is

$$\eta = \eta_{\text{liq}}\left[\exp\left(D\Phi_{\text{wax}}\right) + \frac{E\Phi_{\text{wax}}}{\sqrt{\frac{\partial v_x}{\partial y}}} + \frac{F\Phi_{\text{wax}}^4}{\frac{\partial v_x}{\partial y}}\right] \tag{12}$$

where η_{liq} is the viscosity of the liquid phase with no wax suspended, Φ_{wax} is the volume fraction of solid precipitated, and D, E, F are the fitting parameters.

Pedersen and Rønningsen obtained these parameters by regression of viscosity data from 15 North Sea oils at atmospheric pressure. The API gravity varied from 23.8 to 47.6, the temperatures were within the interval from 273 K to 313 K, and the shear-rate values from 30 s^{-1} to 500 s^{-1} (corresponding to the non-Newtonian shear-thinning region).

In order to apply this model, η_{liq} is obtained by a corresponding states model, tuned to the measured viscosity above the WAT. The volume fraction of wax Φ_{wax} is calculated from the weight fraction and the simulation composition of the precipitated wax, when experimental data are available. If no measured wax precipitation is available, this can be approximated from the procedure also described by Pedersen and Rønningsen.

4 Al-Zahrani Model

The Al-Zahrani model [4] is a model with a power-law shear-thinning region that converges to a plateau at high shear-rate values (the second Newtonian plateau). For the hyperbolic case, the constitutive equation in terms of viscosity is

$$\eta(\dot{\gamma}) = B\left[\left(\frac{\dot{\gamma}+A}{\dot{\gamma}A}\right)^{n} - \frac{1}{\dot{\gamma}^{n}}\right]^{1/n} \tag{13}$$

where the parameter n is tuned to reproduce the degree of shear thinning of the fluid and has values between the Newtonian limit ($n = 1$) up to a value around 3 in the case of oils, A has dimensions of shear rate, B has dimensions of stress, and the ratio B/A tends to the value of viscosity at very high deformation rates (second Newtonian plateau).

5 Power-Law Model

The Ostwald-de Waele equation, more commonly referred to as the power-law model, is one of the simplest models for non-Newtonian viscosity. It can be adequate for shear-thinning (reduction of viscosity with increasing shear rate) and shear-thickening (increment of viscosity with increasing shear rate) behavior when the relationship between log (η) and log ($\dot{\gamma}$) is linear. The model, in terms of viscosity, is

$$\eta = \eta_{\text{ref}}\left(\frac{\dot{\gamma}}{\dot{\gamma}_{\text{ref}}}\right)^{n-1} \tag{14}$$

where η_{ref} is called the consistency parameter, $\dot{\gamma}_{\text{ref}}$ is set to 1 s^{-1}, and n is the behavior index. Three cases must be considered: if $n > 1$, the fluid is dilatant (shear-thickening); $n = 1$ means the fluid is Newtonian; and for $n < 1$, the fluid is pseudoplastic (shear-thinning).

The power-law model cannot be used to represent shear-thinning fluids with any Newtonian plateau. To some degree, however, the power-law model can be used to fit the shear-thinning region by making its parameters temperature dependent. In the case of Oil 1, the following relations are proposed for parameters η_{ref} and n:

$$\eta_{\text{ref}} = \eta_{\text{ref}}^{\infty} 10^{\frac{-a_{\text{k}}}{b_{\text{k}}(1+b_{\text{k}}\exp(c_{\text{k}}T))}} \tag{15}$$

$$n = \frac{a_n}{b_n + \exp(-c_n T)} + d_n \tag{16}$$

Table 3 Oil 1 coefficients for power-law parameter functions

η_{ref}		n	
a_{k}	−0.16780393	a_{n}	0.00458645
b_{k}	0.03985790	b_{n}	0.00627764
$c_{\text{k}}\,(1/^\circ\text{C})$	0.13351848	$c_{\text{n}}\,(1/^\circ\text{C})$	0.21088263
		d_{n}	0.27194431

where T is given in °C and $\eta_{\text{ref}}^{\infty} = 4.80686\,\text{mPa}\cdot\text{s}$, which is the value of η_{ref} in the high-temperature limit. The other parameters are reported in Table 3. Notice that Eq. 16 converges to a value of $n = 1$ at high temperatures; this makes the model to be Newtonian at such temperatures.

Equations 15 and 16 deliver an improved representation of the experimentally observed temperature dependencies of η_{ref} and n. It should be stressed, however, that this temperature corrected model is only applicable within the temperature range of the regression and it cannot be reliably extrapolated too far away from the fitting region.

6 Results

To compare the FT with the other three models, eight of the waxy oils reported by Pedersen and Rønningsen [3] have been tested.

6.1 Pedersen and Rønningsen Model

For the Pedersen and Rønningsen model, it is necessary to know the Φ_{wax} parameter, which is not reported in literature for the waxy oils, except for Oil 4. For this reason, Oil 4 was the only waxy oil available for doing the comparison between the Pedersen and Rønningsen model and the FT model.

Figure 1 shows the overall performance of the Pedersen and Rønningsen model and the FT model for Oil 4. Figure 1b shows how the FT model converges, for zero shear rates, to the first Newtonian region, whereas the Pedersen and Rønningsen model can be seen to diverge. Furthermore, Fig. 1b shows another Newtonian region (the second Newtonian plateau) at high shear rates. Even though Fig. 1a also converges to a Newtonian region for high shear rates, the Pedersen and Rønningsen model cannot fit the viscosity behavior for the full range of temperatures. For the intermediate rate of shearing, the non-Newtonian viscosity thinning behavior is observed for both models. It is important to remark, however, that all the parameters in Eq. 12 were directly fitted to the Oil 4 data in order to optimize, as much as possible, the Pedersen and Rønningsen model performance.

Measured and calculated viscosities for both models were compared as a function of temperature, for three different rates of shearing. The three graphics are shown in Fig. 2a, b, and c. For high shear rates (Fig. 2a), both models yield acceptable predictions

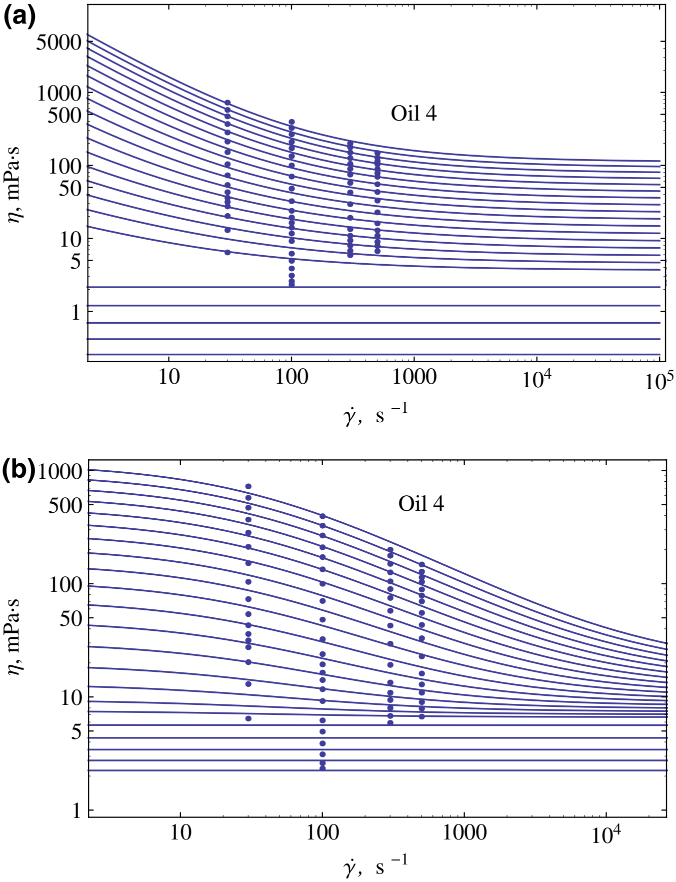

Fig. 1 Overall performance of the (**a**) Pedersen and Rønningsen and (**b**) FT models. The data and curves correspond to several isotherms (353.15, 343.15, 333.15, 323.15, 313.15, 307.15, 305.15, 303.15, 301.15, 299.15, 297.15, 295.15, 293.15, 291.15, 289.15, 287.15, 285.15, 283.15, 281.15, 279.15, 277.15, 275.15) K; the lowest curve and data corresponding to the highest temperature (Newtonian regime) and the highest curve and data corresponding to the lowest temperature (non-Newtonian regime)

of experimental data. However, as the rate of shearing diminishes, the Pedersen and Rønningsen model cannot fit experimental data at low temperatures, while the FT model shows a very consistent performance.

6.2 Al-Zahrani Model

As mentioned in a previous section, the Al-Zahrani model is adequate to represent shear-thinning fluids with a Newtonian region at high values of the shear rate. Furthermore, as B/A tends to this viscosity value, fitting parameter procedures may result in unphysical values of A, B, and n if the second Newtonian region is not evident in the experimental data.

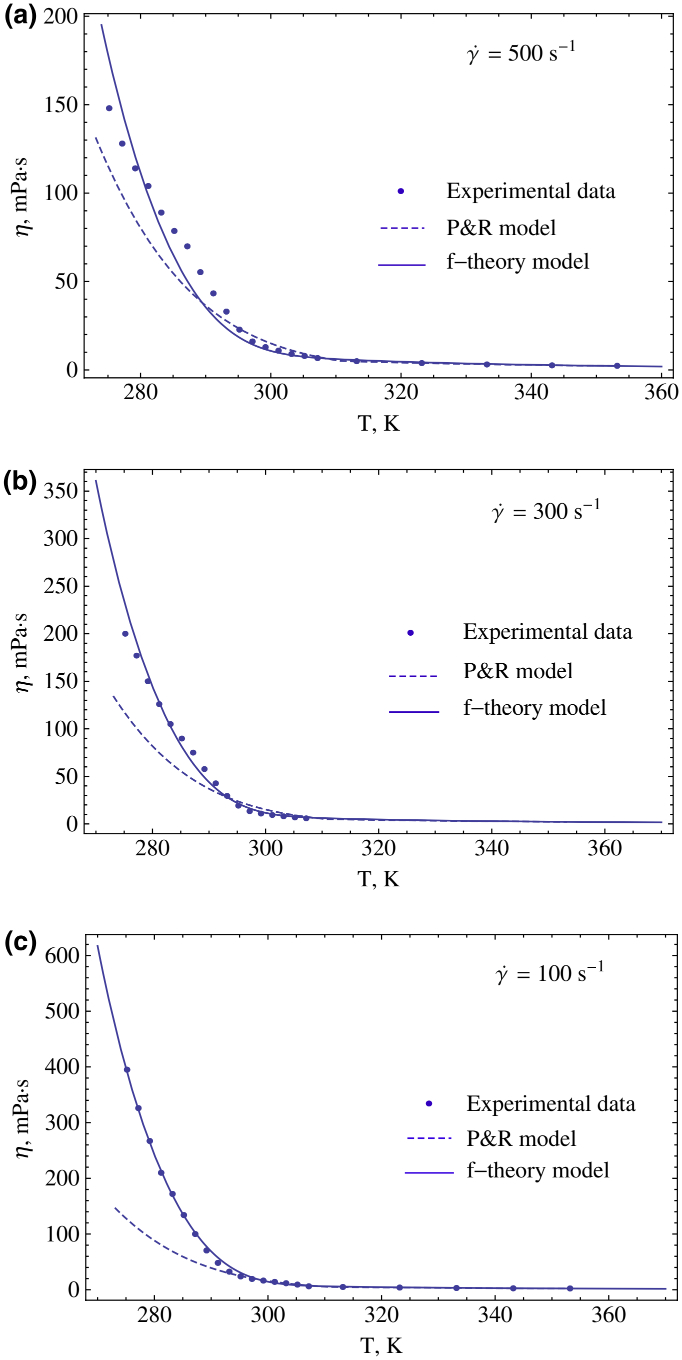

Fig. 2 Measured and calculated viscosity as a function of temperature for three different shear rates: (**a**) 500, (**b**) 300, and (**c**) 100 s^{-1} for FT (*solid line*) and Pedersen and Rønningsen model (*dashed line*)

None of the available data sets exhibit the second viscosity plateau; therefore a direct regression of the Al-Zahrani model parameters to the experimental data is not possible for the cases analyzed in this study.

However, one way to obtain reasonable results with the Al-Zahrani model is by combining this model with a Newtonian model for the representation of the second Plateau. For example, the one-parameter FT model [6] can be used in a predictive mode by assuming that the limiting behavior of the oils converges to that of normal alkanes of similar molar mass. That is, substituting the K_{ref} value given in Eq. 10 in place of the K_{c} in Eq. 3 will result in a prediction for the η_{c} values required in Eq. 2. This predictive model, based on the Newtonian viscosity behavior of normal alkanes [11], can provide a reasonable baseline for the second Newtonian Plateau. That is, since the ratio of B/A of the Al-Zahrani model corresponds to the viscosity of the second Plateau, it is possible to write

$$B = A\eta_{\mathrm{FT}} \tag{17}$$

where η_{FT} is given by the one-parameter FT model, Eq. 1. Yet, in order to obtain reasonable results, it is necessary to make the remaining parameters A and n temperature dependent. Consistent with the values reported by Al-Zahrani [4], for a good representation of the fluids analyzed in this study, the parameter n shows values between two and three at around 275 K which rapidly decrease to a limiting value of one (the Newtonian limit) at about 350 K. A basic model suggestion for the temperature dependency of the n and A parameters is given by the following equations:

$$n = \exp\left[\left(\frac{T_0}{T}\right)^{14}\right] \tag{18}$$

$$A = a_0 \exp\left[\left(\frac{T_0}{T}\right)\right] \tag{19}$$

In the case of Oil 4 the optimal regression values in Eqs. 18 and 19 are $T_0 = 272.515$ K and $a_0 = 4247.74\ \mathrm{s}^{-1}$ which result in an AAD = 41 % compared to a value of 13 % obtained with the FT model (see Table 2). The Oil 4 results of this approach are shown in Fig. 3, where it is clear that the model has the same qualitative properties as in the case of the Pedersen and Rønningsen model, i.e., no first Newtonian plateau is predicted. However, the results are quantitatively better and the model can be easily applied to all the fluids discussed in this study.

6.3 Power-Law Model

Figure 4 shows viscosity measurements for Oil 1. It can be observed that for temperatures lower than the WAT (the non-Newtonian region), viscosity values decrease almost linearly (in logarithmic scales) with increasing deformation rates (shear-thinning response); however, no first or second Newtonian regions are evident. At each temperature, this behavior can be represented by the power-law constitutive equation;

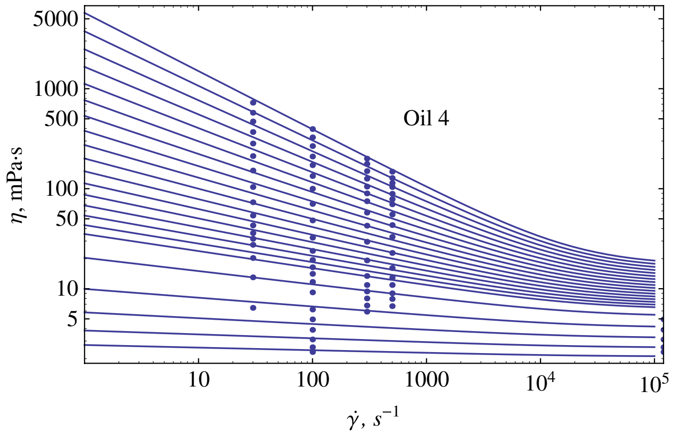

Fig. 3 Overall performance of the Al-Zahrani model for Oil 4. The data for the curves are the same as in Fig. 1

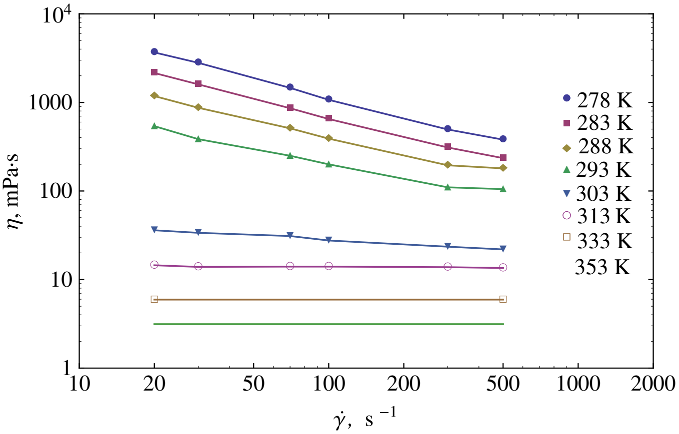

Fig. 4 Viscosity measurements for Oil 1

then parameters η_{ref} and n can be correlated against temperature to obtain a single model for Oil 1 covering the measured temperature range.

The dependence of η_{ref} and n on temperature is plotted in Fig. 5; in this figure, a trend in the power-law parameters is detected, and the parameters in Eqs. 15 and 16 can be fitted to the data showing good results (solid lines). The regression parameters are given in Table 3.

The comparison between the power-law model and the FT model for different shear rates is shown in Fig. 6. It can be observed that both models reproduce the experimental data quite well. The power-law model results in a slightly better approximation; however, it must be remarked that the parameters of the power-law model were obtained by a fitting procedure of Oil 1 data, whereas the FT is a general model which can be utilized for a wide range of systems. In addition, there is no certainty that the

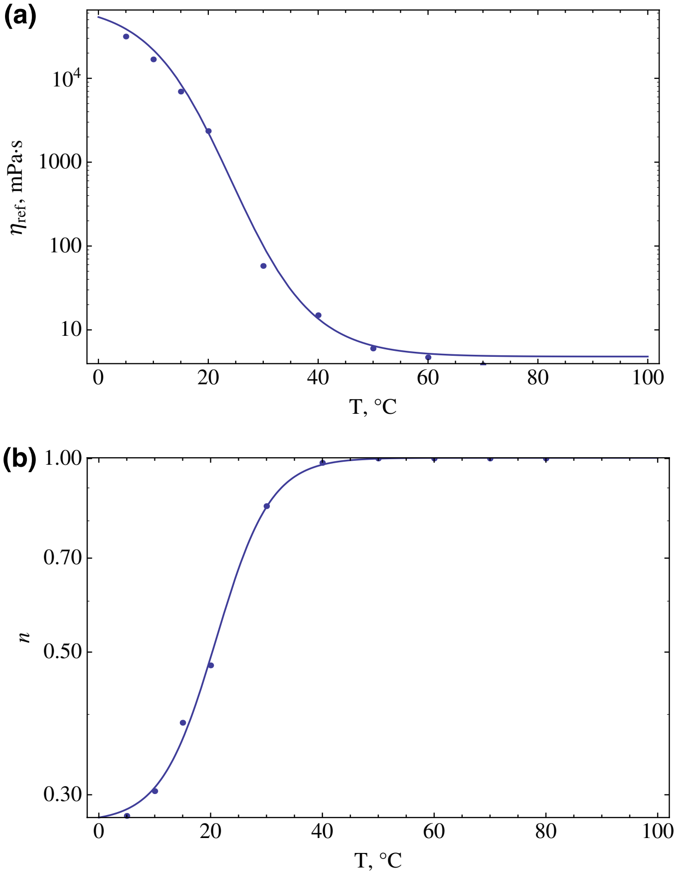

Fig. 5 Power-law parameters at measured temperatures, Oil 1

expressions for η_{ref}—-Eq. 15 and n—Eq. 16, will still be valid for any other system, even if coefficients are recalculated. For example, Fig. 7 shows the resulting n and η_{ref} parameters for Oil 4, where it is evident that Eqs. 15 and 16 are not adequate to reproduce this behavior. In fact, such an abnormal response may be caused by errors in measurements. As the temperature of the fluid increases, the fluid should converge to the Newtonian regime, which means that n must tend to unity in a monotonic fashion. Clearly, given the degree of scattering found in the Oil 4 data, it is almost impossible to obtain an adequate fit to the η_{ref} and n parameters. In contrast, since the physics involved in the transition between the two Newtonian plateaus is implicit in the FT model, the fitting is in essence a scaling approach, which would, at least, result in a correct qualitative behavior—as shown in Fig. 1b for the FT modeling of Oil 4.

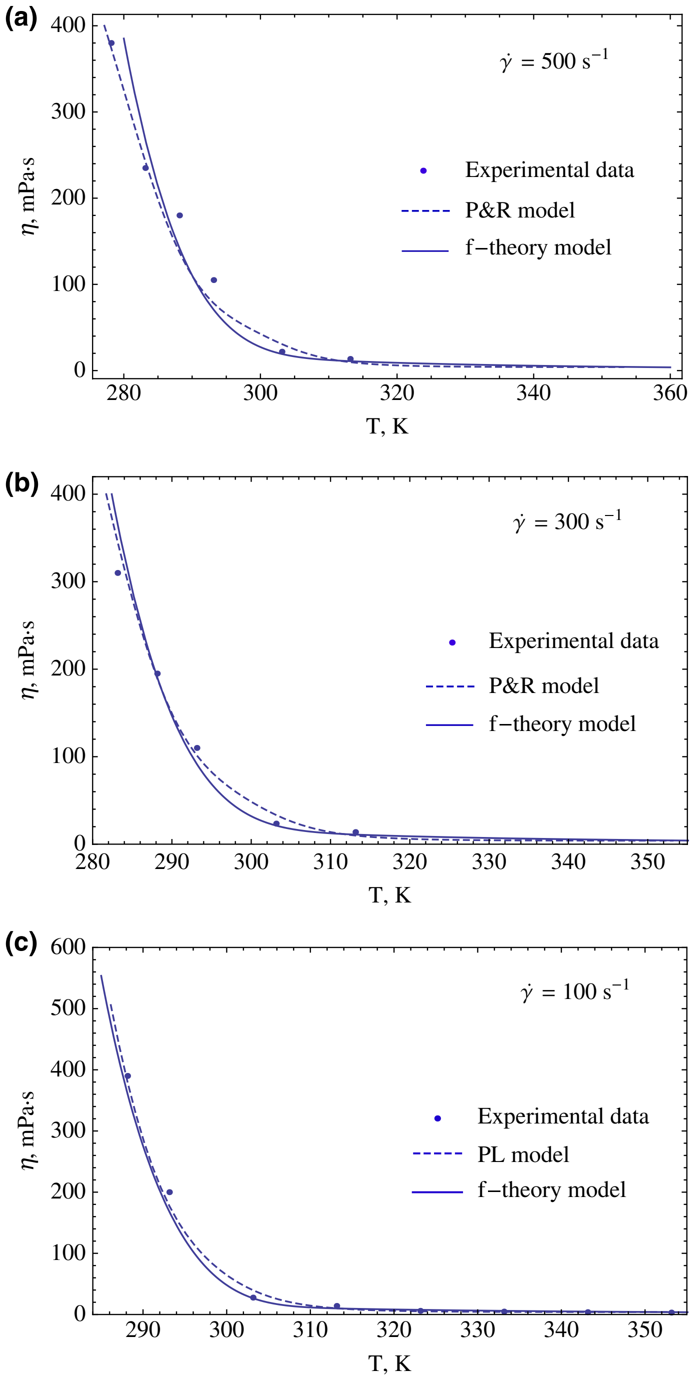

Fig. 6 Oil 1, measured and calculated viscosity as a function of temperature for three different shear rates: (**a**) 500, (**b**) 300, and (**c**) 100 s^{-1} for FT (*solid line*) and Power-Law model (*dashed line*)

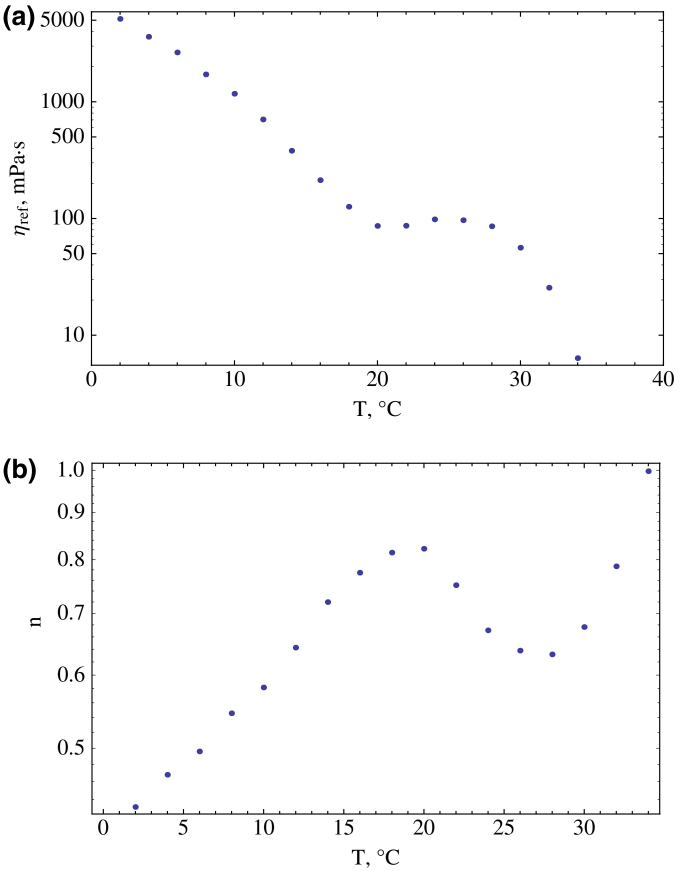

Fig. 7 Power-law parameters at measured temperatures, Oil 4

7 Conclusion

In this study, a comparison among the four models, namely, the FT model, the Pedersen and Rønningsen model, the Al-Zahrani model, and the power-law model is presented.

In the case of the Pedersen and Rønningsen model, the results showed that this model could not reproduce the first Newtonian region due to the intrinsic mathematical limitations of the model. This is not the case for the FT model which can, without affecting the quantitative quality of the results in the viscosity-thinning region, reproduce the two Newtonian regimes. Another result shown is the comparison of viscosity as a function of temperature for different shear rates giving good results in the case of the FT model at any shear rate, but showing significant discrepancies for the Pedersen and Rønningsen model.

In the case of the Al-Zahrani model, it is not possible to directly fit experimental data because the performance of this model depends on the presence of a second Newtonian regime, which is absent in the available experimental data sets. However, combining this model with the FT Newtonian model, it is possible to obtain reasonable results of qualitative similar characteristics to the Pedersen and Rønningsen model but of better quantitative performance and easier application.

The power-law model may accurately reproduce experimental data, provided that they follow a consistent, relatively smooth, trend. This model is easy to use and can deliver fast results; however, it is not a compositional model and a particular analysis for each oil has to be done. In addition, if the measurements are highly scattered (as it is commonly the case for oils) the model may not be able to deliver reliable results.

In conclusion, although several models can reproduce non-Newtonian behavior of waxy crude oils, we have encountered some difficulties in their application. Such difficulties are, in some sense, related to the unbounded nature of the models, i.e., not being able to converge to one of the Newtonian plateaus. The FT model is confined between the two Newtonian plateaus which in some sense bounds the model. Consequently, the FT model can reliably predict non-Newtonian behavior involved in the viscosity-thinning transition between the low and the high shear-rate Newtonian plateaus. In addition, the FT model has the advantage of being a compositional model, which opens numerous application possibilities for the oil industry, among other industrial applications.

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